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Numerical modelling of the passage from free surface to pressurized flow in a closed pipe

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ABSTRACT/RESUME

Abstract: *Urban hydraulic pipelines may be subject to considerable damages while a sudden flow event occurs. However, a transition between free surface and pressurized flow arises; both overpressures and depressions will appear. Controlling this phenomenon becomes a necessity and must be integrated in pipe dimensioning. Most of earlier works were focused on the fictitious piezometric slot. In this work, we aimed to simulate this flow passage as a shock wave and using the Saint Venant mathematical model. Although, in order to take into account the pressurized state; we modified the pressure term. The transition from a type of flow to the other is composed of two discontinues states. Therefore, we solve it as a Riemann problem. To arrive to the most appropriate numerical scheme for the solution, we compare between the results of the Lax Fridricks, lax Wendroff and Godunov schemes. We do this considering the process time, the standard deviation and the Courant Friediricks Levy stability condition. On another hand, we carried out experimental tests, on a transparent and closed circular pipe, to measure pressure change with the flow rate. Hence, we give the physical stationary solution. Finally, we compare numerical results to experimental ones and deduce that the Godunov scheme is the most recommended tool to simulate the flow discontinuity between free surface and pressurized flow*

I. Introduction

During a conduit filling process or rainstorm events, free surface and pressurized flow will exist simultaneously. This is said the mixed flow. In other words, the pipe is partially filled or partially pressurized. A case of a hydroelectric gallery has attracted the attention of Cunge and Wegner [1]. They used the Saint Venant system through the two types of flow considering the Preissman slot approach. They were followed by Dong [2]. Who used the same mathematical model but added the water compressibility and the structure elasticity equations. In 1999, Trajkovic studied the mixed flow using the slot method and a shock capturing model with the explicit McComark scheme. But he was confronted with numerical oscillations [3]. Fuamba [4] divided the mixed flow in three zones and used three different models. For free surface, he used the rigid column model. In the transition section, he used the Saint Venant equations with the characteristic numerical method. He established the interface unknowns by linear interpolation. Then, he used the same mathematical model and takes into account the water compressibility. The air phase composing an important part in this type of flow was considered by Wright and Vasconcelos [5]. Bourdarias and Gerbi introduced the notion of coupled flow and treated the discontinuity as a free limit [6]. Against this Kerger [7] used the Preismann slot approach, but he added the negative one for the depressurized part of the flow. From what is cited below, we deduce that to study the transition between free surface and pressurised flow we must consider the two different flow states and not neglect the depressurized case. Instead of using a single set of free flow equations and considering a slot at the pipe top, we consider the pressure term. Thus, we think that the best manner to simulate more accurately this discontinuity will be by the Riemann Problem approach. We must, also, use the most appropriate capturing shock scheme. Finally, to reach this aim, we are going to compare between solutions of various numerical schemes.

II. Materials and methods

II.1. Mathematical model

The momentum equation combined with the continuity one present the Saint Venant model. It is illustrated as follows:

$$\partial_t U + \partial_x F(x, U) = S(x, U) \quad (1)$$

This is written, under vectored form, as:

$$\vec{U} = \begin{bmatrix} A \\ Q \end{bmatrix}, \vec{F}(\vec{U}) = \begin{bmatrix} Q \\ \frac{Q^2}{A} + p(x, A, E_m) \end{bmatrix}$$

And

$$\vec{S}(\vec{U}) = \begin{bmatrix} 0 \\ gA(S_0 - S_f(x, A, E_m)) \end{bmatrix}$$

Where: *Em* indicates the various states of flow. *p* is the pressure term depending of the position *x*, *A* the flow section and also of the flow state. *Q* is the flow rate. *F* is the flux term. *U is* the unknown vector and *S(U)* represent the source term. g is the gravity acceleration.S_0 is the bottom slope.

II.2. Friction expression

For a pipe with a length of *L*, the friction slope is given by::

$$S_f = \frac{\Delta H}{L} = f \frac{1}{D} \frac{u|u|}{2g} \quad (2)$$

We assume that the flow is fully rough turbulent. So, the Manning Strickler formula will be used but it depends on the flow state.'' S_f '', the friction slope, is given by the following relation:

$$S_f = N_m(A) u|u| \quad (3)$$

Where *u* is the flow velocity and N_m is expressed as follows:

For Free surface flow or a full section no pressurized:

$$N_m(A, E_1) = \frac{n_m^2}{R_h(A)^{\frac{4}{3}}} \quad (4)$$

And for Pressurized or depressurized flow:

$$N_m(A, E_{2,3}) = \frac{n_m^2}{R_h(A_c)^{\frac{4}{3}}} \quad (5)$$

Where: n_m is the Manning coefficient. R_h is the hydraulic radius and *Ac* is the pipe section.

II.3. Pressure expression

In addition to the friction slope, the pressure term is an important element to highlight the discontinuity. So, we have:

For both free and full section flow, we have:

$$p(x, A, E_1) = gAh_c \quad (6)$$

Where h_c designs the water depth from the centre of gravity. It must be evaluated assuming that the internal diameter is equal to the external one.

And for pressurized flow case, pressure is expessed by:

$$\begin{cases} p(x, A, E_2) = gA_c h + c^2(A - A_c) \\ \Delta A > 0 \end{cases} \quad (7)$$

And

$$\begin{cases} p(x, A, E_3) = gA_c h + c^2(A - A_c) \\ \Delta A < 0 \end{cases} \quad (8)$$

This leads to write that the pressure additional term is:

$$h_s = \frac{c^2}{g} \frac{\Delta A}{A_c} \quad (9)$$

Where, *c* is the wave celerity.

The liquid compressibility term generates a problem leading to not permit similarity between the free surface flow celerity and the pressurized flow one. So, we must assume as a hypothesis; an elastic pipe and an incompressible flow. We do this following Vasconcelos [8].

II.4. Numerical modelling

The partially derivative system of equations (1) is hyperbolic and no linear. Consequently, we use the finite volume method. It consists to integer, on simple elementary volumes, equations written as conservative law. It provides a discrete conservative approximation and it is well adapted to the fluid mechanical domain.

Our computational domain is a closed circular pipe, with *L* of length. It is divided on *N* meshes m_i. ($i < I < N$).

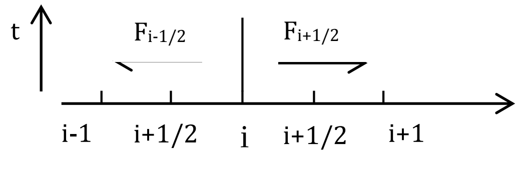

Figure 1. *Pipe discretising*

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With: $m_i = \lfloor x_{i-1/2}, x_{i+1/2} \rfloor$

The correspondent explicit conservative numerical scheme is as follows:

$$U_i^{n+1} = U_i^n - \lambda \left[F_{i+\frac{1}{2}}^n - F_{i-\frac{1}{2}}^n \right] \qquad (10)$$

Where: $\lambda = \frac{\Delta t}{\Delta x}$

This scheme is of one step in time and three steps in space.

Δt is the time step: $t_{n+1} = t_n + \Delta t$

and Δx is the space step : $x_{i+1} = x_i + \Delta x$

The unknown discrete variables of the problem are:

$$U_i^n = \begin{bmatrix} A_i^n \\ Q_i^n \end{bmatrix}$$

II.5. Advection solution

May be useful to solve firstly the following homogeneous system of the partial derivative system:

$$\partial_t U + \partial_x F(x, U) = 0 \qquad (11)$$

This is analogous to a Riemann problem defined as a Cauchy problem with an initial condition composed of two separated states [9], with:

$$U(x, 0) = \begin{cases} U_g \text{ si } x < 0 \\ U_d \text{ si } x > 0 \end{cases}$$

This is for $x \in R$ and $t \in R$ with $t > 0$

If $U(x, t)$ is a weak and an entropic solution of (11), then $U(\lambda x, \lambda t)$ is also a weak and entropic solution.

The advection solution is given by equation (10).

II.6. Source term treatment

Here, we consider the no homogeneous system. The methodology consists of dividing the operation on two steps.

First step: purpose in this step is to give the advection solution:

$$\begin{matrix} U_t + F(U)_x = 0 \\ U(x, t^n) = U^n \end{matrix} \Rightarrow U^{(adv)}$$

second step: ordinary differential equation is given by [10]:

$$\begin{matrix} \frac{dU}{dt} = S(U) \\ U(x, t^n) = U^{(adv)} \end{matrix} \Rightarrow U^{n+1}$$

Thus, solutions obtained from the two previous steps are:

$$U_i^{adv} = U_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2} - F_{i-1/2}\right]$$

And

$$U_i^{n+1} = U_i^{(adv)} + \Delta t S\left[U_i^{adv}\right]$$

Hence, the numerical scheme of solution, taking account the source term S(U), is:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left[F_{i+1/2} - F_{i-1/2}\right] + \Delta t S(U_i^n) \quad (12)$$

II.6. Three space steps schemes

For these types of conservative schemes, we consider three space steps and one time step. They are defined as:

$$U_i^{n+1} = H(U_{i-1}^n, U_i^n, U_i^{n+1})$$

Integrating through; $m_i = \lfloor x_{i-1/2}, x_{i+1/2} \rfloor$; leads to write:

$$\frac{dU_i(t)}{dt} + \frac{1}{\Delta x}\left[F\left(U\left(x_{i+\frac{1}{2}}, t\right)\right) - F\left(U\left(x_{i-\frac{1}{2}}, t\right)\right)\right] = 0$$

Lax friedricks scheme

It is a technique based on decentred finite differentiations in time and in space. Its scheme is presented by:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left(F_{i+1/2}^n - F_{i-1/2}^n\right)$$

With:

$$\begin{cases} F_{i+\frac{1}{2}}^n = \frac{1}{2}(F_{i+1}^n - F_i^n) - \frac{\Delta x}{2\Delta t}(U_{i+1}^n - U_i^n) \\ F_{i-\frac{1}{2}}^n = \frac{1}{2}(F_i^n - F_{i-1}^n) - \frac{\Delta x}{2\Delta t}(U_i^n - U_{i-1}^n) \end{cases} \qquad (13)$$

Lax Wendroff scheme

By means of this scheme, the numerical solution is computed through two phases, as follows [11]:

First phase:

$$U_{i+\frac{1}{2}}^{n+1} = \frac{U_i^n + U_{i+1}^n}{2} - \frac{\Delta t}{2\Delta x}\left(F(U_{i+1}^n) - F(U_i^n)\right) \qquad (14)$$

Second phase:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left(F(U_{i+1/2}^n) - F(U_{i-1/2}^n)\right) \qquad (15)$$

Godunov scheme

This is the exact Riemann solver, it is, also, said 'Flux Difference splitting'. We assume that the solution $''U_i^n''$ is constant on the interval: $\left]x_{i-\frac{1}{2}}\ x_{i+\frac{1}{2}}\right[$. This allows to solve exactly the Riemann problem at each interface and to calculate the solution $''U_i^{n+1}''$.

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left(F\left(U_{i+1/2}^n\right) - F\left(U_{i-1/2}^n\right)\right) \quad (16)$$

With: $U(x,t) = \sum_{k=1}^{2}(x - \lambda_k t, 0)R_k$

The interfacial fluxes are:

$$\begin{cases} F_{i+\frac{1}{2}}^n = \frac{1}{2}\left[F_{i+1}^n + F_i^n - \left|A_{jc}\right|\left(U_{i+1}^n - U_i^n\right)\right] \\ F_{i-\frac{1}{2}}^n = \frac{1}{2}\left[F_i^n + F_{i-1}^n - \left|A_{jc}\right|\left(U_i^n - U_{i-1}^n\right)\right] \end{cases} \quad (17)$$

Knowing that: $A_{jc} = \frac{\partial F(U)}{\partial U}$ is the Jacobean matrix of 'F' depending of 'U'.
Where: $U_i \neq U_{i+1}$

Courant Friediricks Levy condition
Previous numerical schemes must be stable, so they must verify the stability condition of Courant Friediricks Levy [11]:

$$\Delta t = cfl \frac{\Delta x}{\max\{|\lambda_k|\}} \quad (18)$$

With: $0 < cfl < 1$

II.7. Solution computation

Finally, the global solution of the hyperbolic system of derivative partial equation (1) is given by:

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left[\tilde{F}_{I+\frac{1}{2}}^n - \tilde{F}_{i-\frac{1}{2}}^n\right] + \Delta tS(U_i^n)$$

Where; $\tilde{F}_{i+1/2}^n$ and $\tilde{F}_{i-1/2}^n$ are the interfacial Fluxes. In the purpose to solve this equation, characteristics of the computational domain , initial and limit conditions are defined firstly, then the interfacial fluxes and source term are calculated .Results are given for one step of time and three steps of space. A comparison between the numerical schemes will give the most appropriate solution for the domain of application.

II.8. Stationary solution

Experimental setup is composed of a centrifugal pump, a transparent circular closed pipe of three meters length and of 0.05 m diameter. It is equipped of two valves at it's down and up streams. A rectangular tank is used as a source of water. The valves allow us to create transitions in the pipe. An electromagnetic flow meter is used. Manometers are placed at different distances to detect the pressurized sections.

III. Results and discussion

III.1. Results for various courant numbers

Both numerical and exact solution results are presented in figures 2, 3, 4 and 5 in the section, in a square meter, depending of the space position in the meter. The flow rate is illustrated in figures 6, 7, 8 and 9. So, It is the global vector solution $[A \quad Q]$ of the partial derivative equation system modelling the mixed flow occurring inside a closed pipe. It presents the simulation results of the transition discontinuity between two different flow states. This is while we used three numerical schemes: Lax Friedricks, Lax Wendroff and Godunov. For each scheme; we give solutions for various values of the Courant Friedricks Levy number. These values are: 0.1, 0.5, 0.9 and 2.

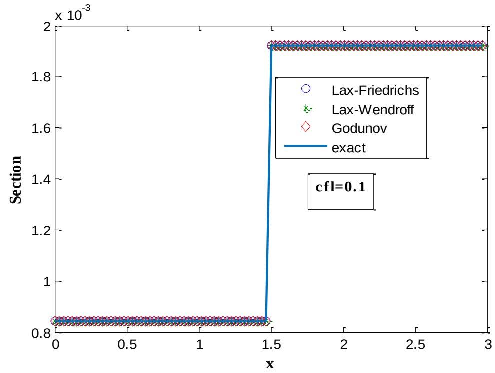

Figure 2. *Numerical solutions of flow section for cfl = 0.1*

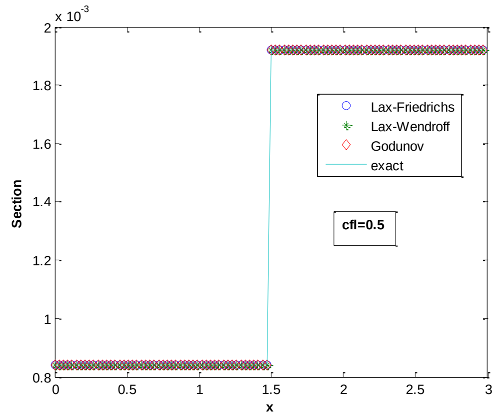

Figure 3. *Numerical solutions of flow section for cfl = 0.5*

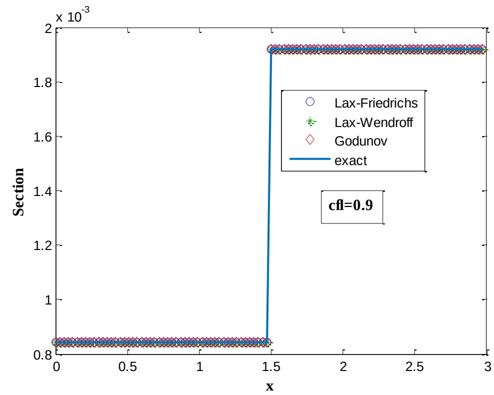

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Figure 4. *Numerical solutions of flow section for cfl = 0.9*

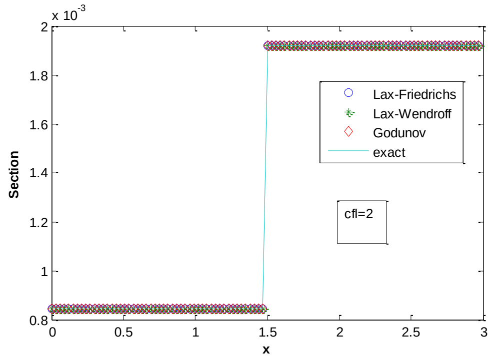

Figure 5. *Numerical solutions of flow section for cfl = 2*

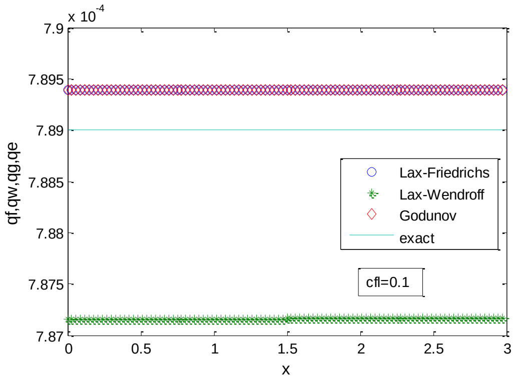

Figure 6. *Numerical solutions of flow rate for cfl = 0.1*

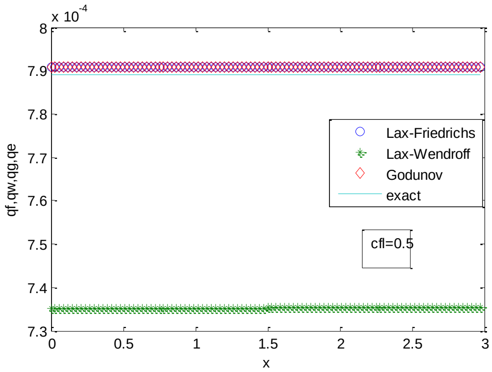

Figure 7. *Numerical solutions of flow rate for cfl = 0.5*

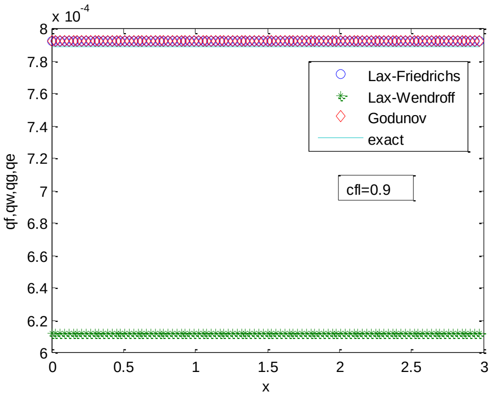

Figure 8. *Numerical solutions of flow rate for cfl = 0.9*

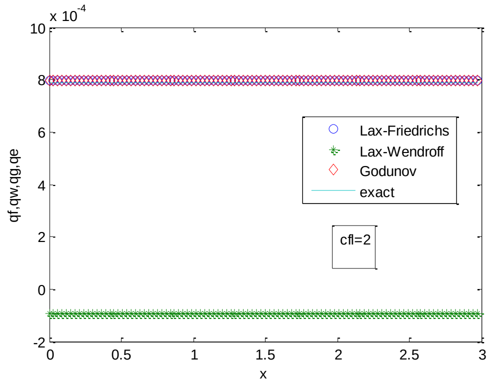

Figure 9. Numerical solutions of flow rate for cfl = 2

III.2. Experimental results discussion

Results issued from experimental measurements are qualified of exact or stationary solution. When flow occurs through the closed pipe a hydraulic jump happens corresponding to a water break, so both first and second conjugate heads were measured. The stationary phenomenon was reached by mean of the up and downstream valves which were carefully manipulated. Furthermore, water body moved expelling air and a considerable difference of the flow height was detected which was also observed by Wright and Vasconcelos[5] and by Chunli[12]. Hence, analogous Piston effect will be a best description of the flow transition while experimental pipe was partially filled.

III.3. Numerical results discussion

Considering smooth material to evaluate the friction slope and assuming an equivalent rectangular section to the circular one, we got the source term expression. Then, the hyperbolic system established previously was solved and all of solutions established, using the three numerical schemes, presented a discontinuity which confirms the piston effect observed during experimentation. The descriptive statistical analysis reveals a maximum section value of 0.00192, a minimum of 0.000842 and a mean of 0.001381, with a standard deviation of 0.0005417. The fraction between the pipe area and each of these values is about 98% for the maximum section, about 43% for the minimum section and about 70% for the mean section. All of these ratios show that the flow will be composed, at the same time, of free and pressurized surface. This is true if we consider the full section at a quotient of 85%.

In order to exam the interface displacement through a considered mesh, we studied four different cases related to Courant number values.

III.4. CPU time and statistic parameters

On another hand of the previous results, tables' n°1 n°2, n°3 and n°4 show the process time designed by CPU time and the statistic parameters of flow results. Which make easier the comparison between the numerical schemes we used in this work The Courant Friediricks Levy number is also an important factor for this comparison.

Table 1. *Schemes comparison for cfl=0.1*

cheme	cpu	std	max	min	mean
Lax Friedricks	248.8528	$2.179*10^{-19}$	0.0009805	0.0009805	0.0009805
Lax Wendrof	248.8840	$4.976*10^{-7}$	0.0004082	0.0004077	0.0004079
Godunov	248.8528	$2.179*10^{-19}$	0.0009805	0.0009805	0.0009805
Exact	experimental	$8.717*10^{-19}$	0.000789	0.000789	0.000789

Table 2. *Schemes comparison for cfl=0.5*

Schéma	cpu	std	max	min	mean
Lax Friedricks	50.4039	$2.179*10^{-18}$	0.00098	0.00098	0.00098
Lax Wendrof	50.3727	$3.76*10^{-7}$	0.0001203	0.0001195	0.0001199
Godunov	50.3727	$1.961*10^{-18}$	0.00098	0.00098	0.00098
Exact	experimental	$8.717*10^{-19}$	0.000789	0.000789	0.000789

Table 3. *Schemes comparison for cfl=0.9*

Scheme	cpu	std	max	min	mean
Lax Friedricks	55.3180	$8.717*10^{-19}$	0.0009771	0.0009771	0.0009771
Lax Wendrof	55.3024	$3.983^{*}10^{-7}$	0.00006929	0.0000685	0.00006889
Godunov	55.3024	$8.717*10^{-19}$	0.0009771	0.0009771	0.0009771
Exact	experimental	$8.717*10^{-19}$	0.000789	0.000789	0.000789

Table 4. *Schemes comparison for cfl=2*

schéma	cpu	std	max	min	mean
Lax Friedricks	14.5081	0	0.0009781	0.0009781	0.0009781
Lax Wendrof	14.4925	-	-	-	-
Godunov	14.5237	0	0.0009781	0.0009781	0.0009781
exact	experimental	$8.717*10^{-19}$	0.000789	0.000789	0.000789

Lax Friedricks and Godunov schemes are characterized by a low value of the standard deviation which means that their simulation results are well distributed. They give same minimal, maximal and mean values as the experimental data. But Godunov scheme has a lower value of the CPU time. However, Lax Wendroff scheme gives a greater value of the standard deviation. Its minimal, maximal and mean values approach half of the exact solution and are less than values of the precedent schemes. Although, for a courant number value of ''0.1'', the CPU time value is more important than the Lax Friedricks and Godunov schemes ones.

When the courant number is upper than one and takes a value of ''2'', which means that the interface moves through two meshes during a time step, Lax Wendroff scheme would not be valid while the two others give a good simulation with a standard deviation of zero.

Considering the previous remarks; Godunov scheme will be the most appropriate and recommended. So, this scheme is the numerical model which is able to describe the transition as a mathematical discontinuity of the flow interface in a partially filled or pressurised pipe. Which looks to be Similar to Kerger's approach pairing Guodunov scheme to both negative and positive piezometric slot model [7].

IV. Conclusion

Transition from free surface flow to pressurized, through closed pipes, present a serious anomaly of hydraulic networks operating. Although, currently, does not exist a well-defined method to remedy to

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this problem. However, transient state resulting from the cited anomaly is governed by a modified Saint Venant partial derivative equations system. So, we add the flow state type in the source term to display the flow transition. Hence, a mathematical discontinuity appears and we solved the governing equations system as a Riemann problem using capturing chock numerical schemes. Among which, we used three ones, for three steps in space and one step in time. On another hand, we carried out experimental tests on a transparent closed pipe in order to present a stationary exact solution.

Descriptive statistical parameters, courant number and process time "CPU", tools we used, to compare the Lax Friedricks, Lax Wendroff and Godunov schemes, allowed us to conclude that:

- Experimental results present the highest value of standard deviation.
- Both Lax Friedricks and Godunov numerical results are distributed better than the Lax Wendroff ones.
- The lowest value of CPU time is detected for the Godunov numerical scheme.
- When, we consider an interface displacement through two meshes in one lap time, the Lax Wendroff scheme is not valid.
- Both experimental and numerical results show an analogous Piston effect of the passage from free surface to pressurized flow.
- We recommend as most appropriate scheme, for computing a partially pressurised transient flow, is the Flux Difference splitting said Godunov scheme.

Finally and wishing expanding the present study background, we envisage to include air phase development and bottom slope. influence on the solution behaviour in future works.

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